NOTE #5 Pag. 56 — **THE TERMINATED lossless T-LINE**

Sept 17 CHAP. 2.3

JUEVES

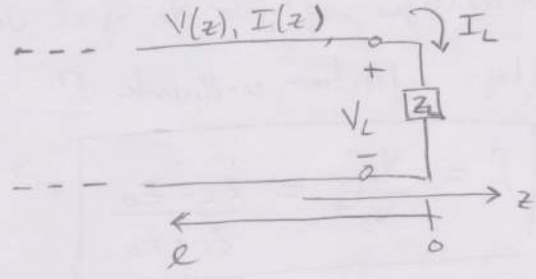


fig 2.4) a transmission line terminated in a load impedance Z_L

→ Asumir q' una incident de la forma $V_0^+ e^{-j\beta z}$ es generada desde una fuente a $z<0$.

→ Hemos visto que la relación de voltage a corriente para esta "traveling wave" es Z_0, que es la impedancia característica de la línea. Sin embargo cuando la línea es terminada en una carga arbitraria $Z_L \neq Z_0$, la relación de voltaje a corriente en la carga debe ser Z_L. Así, una onda reflejada debe poseer una amplitud q' satisfaga dicha condición.

→ El voltaje total en la línea.

$$\boxed{V(z) = V_0^+ e^{-j\beta z} + V_0^- e^{+j\beta z}} \qquad (2.3.4a)$$

→ la corriente total

$$\boxed{I(z) = \frac{V_0^+}{Z_0} e^{-j\beta z} - \frac{V_0^-}{Z_0} e^{+j\beta z}} \qquad (2.3.4b)$$

→ En la carga $z=0$

$$Z_L = \frac{V(0)}{I(0)} = \frac{V_0^+ + V_0^-}{\frac{V_0^+ - V_0^-}{Z_0}} = \boxed{\frac{V_0^+ + V_0^-}{V_0^+ - V_0^-} \cdot Z_0} = Z_L$$

− resolviendo para V_0^-: $(V_0^+ + V_0^-) Z_0 = Z_L (V_0^+ - V_0^-)$

$$Z_0 V_0^- + Z_L V_0^- = Z_L V_0^+ - Z_0 V_0^+ \Rightarrow V_0^- (Z_0 + Z_L) = V_0^+ (Z_L - Z_0)$$

$$\boxed{V_0^- = \frac{Z_L - Z_0}{Z_L + Z_0} \cdot V_0^+}$$

→ La relación entre el voltaje reflejado V_0^- y el voltaje incidente V^+ se conoce como voltage reflection coefficiente Γ

$$\boxed{\Gamma = \frac{V_0^-}{V_0^+} = \frac{Z_L - Z_0}{Z_L + Z_0}} \rightarrow \boxed{V_0^- = \Gamma V_0^+} \quad (2.35)$$

→ las ondas de voltaje y corriente en la línea pueden re-escribirse.

$$V(z) = V_0^+ e^{-j\beta z} + V_0^- e^{+j\beta z}$$

$$V(z) = V_0^+ e^{-j\beta z} + \Gamma V_0^+ e^{+j\beta z}$$

$$\boxed{V(z) = V_0^+\left[e^{-j\beta z} + \Gamma e^{+j\beta z}\right]} \quad (3.36a)$$

$$I(z) = \frac{V_0^+}{Z_0^+} e^{-j\beta z} - \frac{V_0^-}{Z_0^-} e^{+j\beta z}$$

$$= \frac{V_0^+}{Z_0^+} e^{-j\beta z} - \frac{\Gamma}{Z_0} V_0^+ e^{+j\beta z}$$

$$\boxed{I(z) = \frac{V_0^+}{Z_0}\left[e^{-j\beta z} - \Gamma e^{+j\beta z}\right]}$$

– Podemos ver q' el V y la I en la línea consiste de una superposición de una onda incidente y una onda reflejada.

→ a dicha onda se le conoce como: standing wave.

→ cuando $\Gamma = 0$ → no hay onda reflejada.

→ Para obtener $\Gamma = 0$, $Z_L = Z_0$ → condición "matched load"

Consideremos el Time-average-power along the line

$$\boxed{P_{avg} = \frac{1}{2}\frac{|V_0^+|^2}{Z_0}\left(1-|\Gamma|^2\right)} \qquad 2.3.7$$

Total power delivered to the load: incident power $\frac{|V_0^+|^2}{2Z_0}$ – minus the reflected Power $\left(\frac{|V_0|^2|\Gamma|^2}{2Z_0}\right)$.

→ $\Gamma = 0$ → max power delivered to the load.

$\Gamma = 1$ → No power delivered to the load.

→ Return Loss: $RL = -20\log|\Gamma|\ dB$

$\Gamma = 0 \longrightarrow RL = \infty\ dB$ (No reflected Power)

$|\Gamma| = 1 \longrightarrow RL = 0\ dB$. (all incident power is reflected)

→ Máximo voltage en la línea

$$\boxed{V_{max} = |V_0^+|(1+|\Gamma|)}$$

→ Mínimo Voltage en la línea

$$\boxed{V_{min} = |V_0^+|(1-|\Gamma|)}$$

→ Standing Wave Ratio (voltage standing wave ratio).

$$\boxed{SWR = \frac{V_{max}}{V_{min}} = \frac{1+|\Gamma|}{1-|\Gamma|}}$$

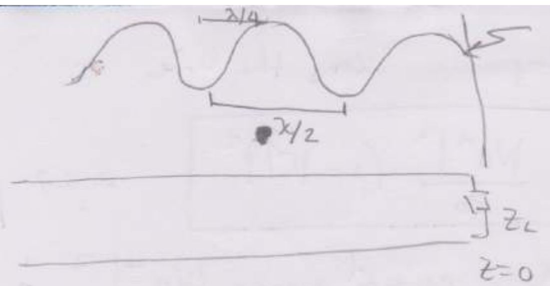
$1 \leq SWR \leq \infty$

$SWR = 1$ → matched load.

→ distance entre dos maximos de voltaje (o minimos)

$$\boxed{l = \lambda/2}$$

→ distancia entre un maximo y un minimo: $\boxed{l = \lambda/4}$



→ Reflected coefecient en cualquier punto de la linea

$$\Gamma(\ell) = \frac{V_0^- e^{-j\beta\ell}}{V_0^+ e^{j\beta\ell}} = \Gamma(0)\, e^{-2j\beta\ell}$$

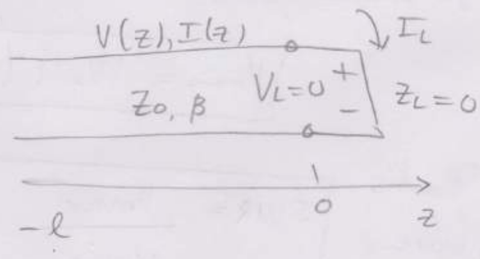
$\Gamma(0)$ = coeficiente de reflexión at $z=0$ (load)

→ Transmission line impedance equation:

$$\boxed{Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan\beta\ell}{Z_0 + jZ_L \tan\beta\ell}}$$

⇒ Special Cases of lossless T-Line

a) $Z_L = 0$ short circuit



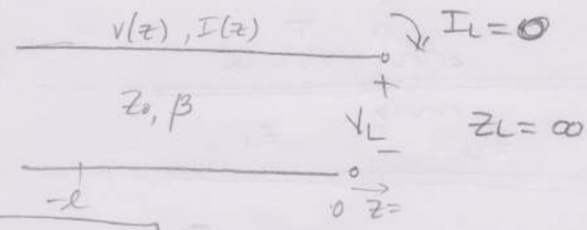
$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = -1$$

$$Z_{in} = Z_0 \frac{0 + jZ_0 \tan\beta\ell}{Z_0 + j(0)\tan\beta\ell} = \boxed{jZ_0 \tan\beta\ell} \qquad (2.45c)$$

when $\ell = 0 \rightarrow Z_{in} = 0$

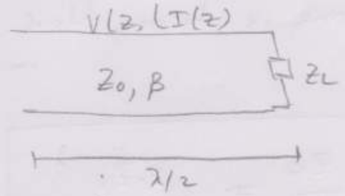
$\ell = \lambda/4 \rightarrow Z_{in} = \infty$ (open circuit)

b) OPEN CIRCUIT CASE



$$\boxed{Z_{in} = -j Z_0 \cot \beta \ell}$$

— LENGTH $\lambda/2$



$$\beta \ell = \frac{2\pi}{\cancel{\lambda}} \cdot \frac{\cancel{\lambda}}{2} = \pi$$

$$Z_{in} = Z_0 \frac{Z_L + j Z_0 \tan \beta \ell}{Z_0 + j Z_L \tan \beta \ell} = Z_0 \frac{Z_L + j Z_0 \cancel{\tan(\pi)}^{0}}{Z_0 + j Z_L \cancel{\tan(\pi)}^{0}} = Z_L$$

$$\boxed{Z_{in} = Z_L}$$

— LENGTH $\lambda/4$

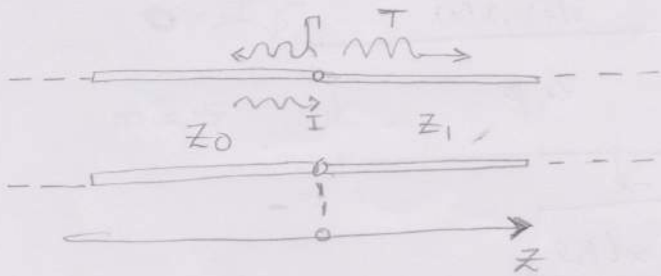
$$\beta \ell = \frac{2\pi}{\cancel{\lambda}} \cdot \frac{\cancel{\lambda}}{4} = \pi/2$$

$$Z_{in} = Z_0 \frac{Z_L + j Z_0 \tan \beta \ell}{Z_0 + j Z_L \tan \beta \ell} = Z_0 \frac{Z_L + j Z_0 \tan(\pi/2)}{Z_0 + j Z_L \tan(\pi/2)}$$

$$Z_{in} = Z_0 \frac{\cancel{\tan(\pi/2)}}{\cancel{\tan(\pi/2)}} \left[\frac{\cancel{Z_L / \tan \pi/2}^{0} + j Z_0}{\cancel{Z_0 / \tan \pi/2}^{0} + j Z_L} \right] = Z_0 \cdot \frac{j Z_0}{j Z_L}$$

$$\boxed{Z_{in} = \frac{Z_0^2}{Z_L}}$$ (quarter wave transform). // class on Tuesday

JUEVES 24-sept , Quiz # 3.



$$\boxed{\Gamma = \frac{Z_1 - Z_0}{Z_1 + Z_0}}$$

Not all incident wave is reflected; some is transmitted

$$\boxed{T = 1 + \Gamma = 1 + \frac{Z_1 - Z_0}{Z_1 + Z_0} = \frac{2Z_1}{Z_1 + Z_0}}$$

Transmission coefficient

$$\boxed{I_L = -20 \log |T| \ dB}$$

insertion Loss.